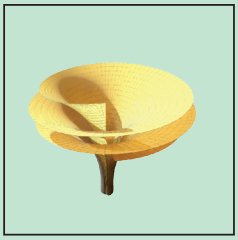

XXI
GEOMETRICAL SEMINAR

Book of Abstracts

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The matrices of Pauli quaternions, their De Moivre's formulas

Mücahit Akbiyik[1]

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In this study, we obtain De Moivre's formulas for the matrix representation of Pauli quaternions. Additionally, we give the n^{th} roots of the matrix representation of Pauli quaternions.

Clifford algebras and homogeneous convex cones: applications information geometry and supergravity

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The following subjects will be discussed.
1. Graded Clifford algebras and graded Clifford modules.
2. Geometry of homogeneous convex cones.
3. Information geometry and homogeneous convex cones.
4. Vinberg theory of homogeneous convex cones. Isometric maps and Clifford modules.
5. Selfdual cones and Euclidean Jordan algebras Special Vinberg cones associated to graded Clifford modules.
6. Application of rank 3 special Vinberg cones to $N = 2$ Supergravity

in dimension $d = 5, 4, 3$.
7. Application of rank 3 special Vinberg cones to calculation of entropy of BPS black holes in $N = 2$ $d = 4$ Supergravity.

Proportionality in Riemannian Osserman manifolds

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We present some recent and new results related to Riemannian Osserman manifolds. We introduce the notion of the Jacobi-proportional algebraic curvature tensor as an (Osserman) algebraic curvature tensor with the property that for any two unit vectors X and Y, the projections of X to the eigenspaces of the Jacobi operator $\mathcal{J}_Y$ have the same lengths as the projections of Y to the corresponding eigenspaces of the Jacobi operator $\mathcal{J}_X$. This extremely non-trivial algebraic property appears to be quite natural in the context of the Osserman property, and can be viewed as a development of the Rakić duality principle. The Jacobi-proportionality can be very effective in the study of the Osserman conjecture, while based on the results of Nikolayevsky, we prove that Jacobi-proportional Osserman tensors that do not satisfy the Osserman conjecture are 2-root with multiplicities 8 and 7, or 3-root with multiplicities 7, 7, and 1.

Functions harmonic and $\mathcal{M}$-harmonic in the unit polydisc

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We give a complete characterization of functions which are at the same time harmonic and $\mathcal{M}$ - harmonic in the unit polydisc. As a consequence we obtain a characterization of functions u such that both u and u^s ($s \geq 2$) have this property. Our results stand in marked contrast to results known for such functions defined on the unit ball in $\mathbb{C}^n$. This is a joint work with Jelena Gajic.

Horospherical coordinates and curvature properties of complex hyperbolic space equipped with different left-invariant metrics

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Complex hyperbolic space is a non-compact rank-one symmetric space of negative sectional curvature. Thus, it can be represented as a connected solvable real Lie group with a left-invariant metric. This group is a semidirect product of the abelian and nilpotent part (Heisenberg group) of the Iwasawa decomposition of its isometry group: $\mathcal{CH}^n = \mathbb{R} \ltimes H^{2n-1}$. Its structure is beautifully seen in the Siegel paraboloid model of $\mathbb{C}H^n$ in $\mathbb{C}P^n$, with "horospherical coordinates" first introduced by Goldman and Parker [1]. All possible left-invariant metrics on this Lie group have been classified recently [2]. We search for geodesics associated with these metrics in the special 4-dimensional case $\mathcal{CH}^2$. Using the Euler-Arnold equation one can simplify the system of 2nd-order differential equations of geodesics on a Lie group to a system of 1st-order differential equations on its Lie algebra. By solving these equations numerically, we visualize geodesic spheres.

1. W. Goldman, J. Parker, *Dirichlet Polyhedra for Dihedral Groups Acting on Complex Hyperbolic Space*, The Journal of Geometric Analysis Volume 2, Number 6, 1992
2. A. Dekić, Marijana Babić, Srdjan Vukmirović, *Classification of Left Invariant Riemannian Metrics on Complex Hyperbolic Space*, Mediterranean Journal of Mathematics, to appear 2022.

Modified gravity with the Gauss-Bonnet invariant

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Within the plethora of modified theories of gravity, particular interest is gained by modifications including the Gauss-Bonnet topological surface term. Extending the gravitational action with the Gauss-Bonnet invariant leads to interesting dynamics in cosmological and spherically symmetric backgrounds. Specifically, the early and late-time Universe bahaviours can be predicted without invoking any Dark Energy, which is thus mimicked by geometric contributions. Moreover, considering the Gauss-Bonnet term into the action, can settle theoretical issues, actually suffered by General Relativity. After selecting the form of the action, the gravitational field equations can yield analytic solutions whose free parameters can be constrained by experiments and observations. Specifically, the starting model can be selected by the so called "Noether Symmetry Approach", a selection criterion aimed at finding easily-handled models containing symmetries. The resulting conserved quantities can be used to reduce the minisuperspace dimension and find out exact solutions. In this presentation, different modifications of the Einstein-Hilbert action including the Gauss-Bonnet term are discussed, such as scalar-tensor models, non-local models, extended and teleparallel models.

Invariant structures on homogeneous Φ-spaces and Lie groups

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Homogeneous Φ-spaces were first introduced by V.I. Vedernikov in 1964. Fundamental results for, in particular, homogeneous k-symmetric spaces were obtained by N.A. Stepanov, A. Ledger, A. Gray, J.A. Wolf, A.S. Fedenko, O. Kowalski and others. It turned out that homogeneous k-symmetric spaces G/H admit a commutative algebra $\mathcal{A}(\theta)$ of *canonical* structures (the author and N.A. Stepanov). The classical example is the canonical almost complex structure J on homogeneous 3-symmetric spaces with its many applications (N.A. Stepanov, A. Gray, V.F. Kirichenko, S. Salamon and others). For $k > 3$ the algebra $\mathcal{A}(\theta)$ contains a large family of classical structures such as almost complex ($J^2 = -id$), almost product ($P^2 = id$), f-structures of K. Yano ($f^3 + f = 0$) and some others. We dwell on several applications of canonical structures as well as on left-invariant structures on nilpotent and solvable Lie groups.

1) *The generalized Hermitian geometry* (V.F. Kirichenko, D. Blair, S. Salamon and others): canonical nearly Kähler, Killing, Hermitian metric f-structures on homogeneous k-symmetric spaces; left-invariant nearly Kähler and Hermitian f-structures on some classes of nilpotent Lie groups (especially, on 2-step nilpotent and some filiform Lie groups); on generalized (in various senses) Heisenberg groups in dimension 5, 6, and 8; on solvable Lie groups in dimension 3 and 4.

2) *Homogeneous Riemannian geometry*: the Naveira classification of Riemannian almost product structures; canonical distributions on Riemannian homogeneous k-symmetric spaces; the classes **F** (foliations), **AF** (anti-foliations), **TGF** (totally geodesic foliations); the Reinhart foliations.

3) *Elliptic integrable systems*: homogeneous k-symmetric spaces and associated elliptic integrable systems; a new generalization of almost Hermitian geometry; a new contribution to nonlinear sigma models (F. Burstall, I. Khemar).

4) *Symplectic geometry*: bi-Poisson geometry and bi-Hamiltonian systems (A.V. Bolsinov, A.M. Izosimov, D.M. Tsonev), Hamiltonian vector fields and integrable almost-symplectic Hamiltonian systems (F. Fasso, N. Sansonetto), canonical almost symplectic structures on Riemannian homogeneous k-symmetric spaces.

Random moment angle complex

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Let n be a positive integer and $p \in [0,1]$. The *random simplicial d-complex* $Y^d_{n;p}$ on n vertices and with parameter p, introduced by Meshulam and Wallach, has the following probability law:
- $Y^d_{n;p}$ takes values in the space of all simplicial complexes K such that $\Delta^n_{d-1} \subset K \subset \Delta^n_d$
- each possible d-simplex of Δ^n_d appears in $Y^d_{n;p}$ with probability p, independently.
We introduce moment angle complex over the random simplicial complex and study its topological and combinatorial features. Specially, some Law of the large numbers for the bigraded Betti numbers has been established. Several other directions for studying randomness in toric topology have been considered.

Generic submanifolds of almost contact metric manifolds

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Ronsse introduced the notion of generic and skew CR-submanifolds of almost Hermitian manifolds to unify and generalize the notions of holomorphic, totally real, CR, slant, semi-slant and pseudo-slant submanifolds. Other authors, as Tripathi extended this notion to contact geometry, under the name of almost semi-invariant submanifolds. This class includes the one with the same name introduced by Bejancu, (and studied also by Tripathi) but without being equal. The class of submanifolds that we introduce and study here in contact geometry, is called by us generic submanifolds, to avoid the above confusion, and since it is more general from the class studied by Tripathi. We obtain necessary and sufficient conditions for the integrability and parallelism of some eigen distributions of a canonical structure on generic submanifolds. Some properties of the Reeb vector field to be Killing and its curves to be geodesics are investigated. Totally geodesic and mixed geodesic results on generic submanifolds are established. We give necessary and sufficient conditions for a generic submanifold to be written locally as a product of the leaves of some eigen distributions. Some examples on both generic submanifolds and skew CR-submanifolds of almost contact metric manifolds are constructed.

The space of centered planes and generalized bilinear connection

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We continue to study the space of centered planes in n-dimension projective space. We use E. Cartan's method of external forms and the group-theoretical method of G.F. Laptev to study the space of centered planes of the same dimension. These methods are successfully applied in physics.

In a generalized bundle, a bilinear connection associated with a space is given. The connection object contains two simplest subtensors and subquasi-tensors (four simplest and three simple subquasi-tensors).

The object field of this connection defines the objects of torsion S, curvature-torsion T, and curvature R. The curvature tensor contains six simplest and four simple subtensors, and curvature-torsion tensor contains three simplest and two simple subtensors.

The canonical case of a generalized bilinear connection is considered.

Integrable billiards and its applications

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Currently, integrable Hamiltonian systems with two degrees of freedom are being actively studied.

If H and f are the first integrals of such a system, then we can fix a non-singular value c of the function H and see what foliation the function f defines on the isoenergy surface $(H = c)$. In this case, two IHSs are considered (Liouville) equivalent if there exists a fiberwise homeomorphism of corresponding foliations. A. T. Fomenko and H. Zieschang constructed an invariant of such an equivalence — a labeled molecule.

A very important class of integrable systems are billiards bounded by arcs of confocal quadrics. In the case of two degrees of freedom, the additional first integral of such billiards is the parameter of caustic.

Not so long ago, V. V. Vedyushkina discovered a new class of billiards — billiard books. This is when several billiard tables are glued together along common ribs-backs. This approach significantly extended the class of integrable billiards.

Recently G. V. Belozerov has been actively studying billiards with three degrees of freedom. A weaker equivalence relation is introduced on the set of such systems, and a classification theorem for billiards restricted to confocal quadrics is proved. A complete classification of billiard «tables» with respect to their combinatorial equivalence is obtained.

Facet volumes of polytopes

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In this lecture, we show that for fixed integers $d \geq 2$ and $n \geq d+1$ the configuration space of all facet volume vectors of all d-polytopes in $\mathbb{R}^d$ with n facets is a full dimensional cone in $\mathbb{R}^n$. In particular, for tetrahedra ($d = 3$ and $n = 4$) this is a cone over a regular octahedron. Our proof is based on a novel configuration space / test map scheme which uses topological methods to find solutions of a problem, and tools of differential geometry to identify solutions with the desired properties. (The lecture is based on a joint work with Paul Breding and Alexander Heaton.)

Rarefied cloud contractibility

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It is shown that the so-called rarefied clouds are contractibile in themselves to a point, which provides a partial proof of Gromov's remark about the contractibility of any cloud.

Bounds on graviton mass from planetary motion in the Solar System

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In this study we used the observed perihelion precession of planetary orbits in the Solar System in order to constrain several theories of Yukawa-type gravity, and to bound mass of graviton. For that purpose we assumed that the precession angles of the planets in Yukawa-type gravity should be equal to their observed values, as well as to the corresponding predictions of General Relativity (GR). The obtained results showed that Yukawa-type gravity can induce the angles of orbital precession which could be very close to their observed values, or to the values predicted by GR. Assuming that the estimated value of the range of Yukawa interaction Λ corresponded to the Compton wavelength of graviton λ_g, we then estimated the upper bound for its mass m_g. We found that these estimates were in agreement with our previous results obtained from the observed stellar orbits around the Galactic Center (GC).

Smooth metric measure spaces and the Einstein equation

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A semi-Riemannian manifold (M, g) equipped with a density function f gives rise to a smooth metric measure space $(M, g, e^{-f} dvol_g, \mu)$, where μ is an extra parameter with a geometric meaning. Usual curvature tensors such as the Ricci tensor, the Weyl tensor or the Schouten tensor are generalized in these structures by *weighted* tensors which include information on the density function and also involve the parameter μ. Taking as a starting point the Bakry-Émery Ricci tensor $\rho^f = \rho + \mathrm{Hes}_f - \mu df \otimes df$ we study a generalization of the Eintein equation.

In a Lorentzian smooth metric measure space, motivated by characterizing properties of the usual Einstein tensor, we define a weighted Einstein tensor. As a first step, we study the resulting vacuum Einstein field equation, with special emphasis on the isotropic case (i.e. ∇f is a null vector field). We describe the Ricci tensor of isotropic solutions in general, which are classes of Kundt spacetimes. In dimension 3 we classify solutions given a description in local coordinates.

The gravitational energy-momentum pseudotensor in higher-order theories of gravity

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We derive the gravitational energy momentum tensor for a general Lagrangian of any order and prove that this tensor, in general, is not covariant but only affine, then it is a pseudo-tensor. Furthermore, the pseudo-tensor is calculated in the weak field limit up to a first non-vanishing term of second order in the metric perturbations. The average value of the pseudo-tensor over a suitable spacetime domain is obtained. Finally we calculate the power per unit solid angle carried by a gravitational wave. These results are useful in view of searching for further modes of gravitational radiation beyond the standard two modes of General Relativity and to deal with non-local theories of gravity. The general aim of the approach is to deal with theories of any order under the same standard.

Helicoid and curvature based functional variations

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In view of the meaning of ruled surfaces in aesthetics, statics, scale and manufacturing technologies, we point out the possibility of a mathematical analysis in the case of infinitesimal deformations by considering the rigidity of surfaces. In this work we consider variations of the shape operator, the normal curvature and the principal curvatures of helicoid under infinitesimal bending.

AMS Subj. Class.: 53A05, 53C45, 92C40

Key words: Shape operator, Helicoid, Infinitesimal bending, Variation.

On hybrid quaternions

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In this study, we firstly define a new quaternion system called hybrid quaternions by taking the coefficients of real quaternions as hybrid numbers. This new type of quaternion system is a combination of complex quaternions (biquaternions), dual quaternions, and hyperbolic (perplex) quaternions, and it can be regarded as a generalization of these quaternion systems. We then present the fundamental properties of hybrid quaternions including fundamental operations, conjugates, inner product, vector product, and norm. Finally, we give a diagrammatic representation of numbers and quaternions in order to explain the relations of number systems.

On the centralizer rigidity phenomenon

Danijela Damjanović

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I will describe the phenomenon which occurs for certain class of partially hyperbolic automorphisms on nilmanifolds. Such maps have symmetry groups which are easy to compute. Then we consider small perturbations of such maps and we find that the smooth centralizer of any perturbation falls into one of the finitely many classes. Moreover, if it is isomorphic to the centralizer of the unperturbed map we obtain certain rigid properties of the perturbation. This is joint work with Amie Wilkinson and Disheng Xu.

Classification of left invariant Riemannian metrics on complex hyperbolic space

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It is well known that $\mathbb{C}H^n$ has the structure of solvable Lie group with left invariant metric of constant holomorphic sectional curvature. In this talk, we give the full classification of all possible left invariant Riemannian metrics on this Lie group. We fix Lie algebra commutators, then consider the space of all possible left invariant metrics, and find the simplest representatives under the action of the automorphism group. The isometry classes of left invariant metrics are exactly the orbits of the automorphism group acting on the space of left invariant metrics.

Chen type of spherical hypersurfaces via the second standard immersion of the sphere

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Some of the most interesting examples of spherical hypersurfaces are isoparametric hypersurfaces that have constant principal curvatures whose number can be only among the values $g = 1, 2, 3, 4,$ and 6. They have been largely classified, except the ones with 4 principal curvatures which comprise a rather plentiful set of examples. In our previous published work, the type of minimal isoparametric hypersurfaces with 2 and 3 principal curvatures have been studied, via the immersion by projection operators. We will treat here the spherical isoparametric hypersurfaces which are of low type via the second standard immersion of the sphere in the Euclidean space of symmetric matrices. These include Chen-type of non-minimal cubic isoparametric hypersurfaces and some examples with 4 curvatures, among them the Cartan-Nomizu example in S^5.

Cosmological solutions of a nonlocal gravity model

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A nonlocal gravity model

$$S = \frac{1}{16\pi G}\int (R - 2\Lambda + (R-4\Lambda)\mathcal{F}(\Box)(R-4\Lambda))\sqrt{-g}\mathrm{d}^4x$$

was recently introduced, and two exact cosmological solutions in flat space were presented.

The first solution has similar properties to an interference between the radiation and the dark energy, while the second one is a nonsingular time symmetric bounce. In this talk we investigate other possible exact cosmological solutions and find some new ones in nonflat space. Used nonlocal gravity dynamics can change background topology. To solve the corresponding equations of motion, we first look for a solution of the eigenvalue problem $\Box(R-4\Lambda) = q(R-4\Lambda)$. We also discuss possible extension of this model with nonlocal operator symmetric under $\Box \longleftrightarrow \Box^{-1}$ and its connection with another interesting nonlocal gravity model.

This talk is based on joint work with Branko Dragovich, Zoran Rakić and Jelena Stanković.

Chebyshev dynamics, isoharmonic deformations, and constrained Schlesinger systems

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The talk is based on interrelations between integrable billiards, extremal polynomials, Riemann surfaces, potential theory, and isomonodromic deformations. We discuss injectivity properties of rotation and winding numbers. We study dynamics of Chebyshev polynomials on several intervals and introduce a notion of iso-harmonic deformations. We study their isomonodromic properties and formulate a new class of constrained Schlesinger systems. We provide explicit solutions to these systems. The talk is based on joint results with Vasilisa Shramchenko, including work in progress.

The sixth power moment of Dirichlet L-functions over rational function fields

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The distribution of primes is determined by the distribution of zeros of Dirichlet L-functions, and it is indirectly determinated by the distribution of magnitude of these functions on the critical line $\Re s = \frac{1}{2}$. Therefore, one of the main goals in Analytic Numeber Theory is to consider the moments of Dirichlet L-functions (according to a certain well defined family). It is still an open question except for the first few moments, and it is related to the Generalized Riemann and Lindelöf Hypothesis. Random Matrix Theory provides hypotheses about the moments of L-functions.

In this talk we consider analogous questions over rational function fileds $\mathbb{F}_q(t)$, where $\mathbb{F}_q$ is a finite field. We will present the asymptotic formula for the sixth power moment

$$\sum_{\substack{Q \text{ monic} \\ \deg Q = \mathbf{d}}} \sum_{\substack{\chi \pmod Q \\ \chi \text{ odd} \\ \text{primitive}}} \int_0^{\frac{2\pi}{\log q}} \left| L\left(\frac{1}{2} + it, \chi\right) \right|^6 \frac{dt}{2\pi/\log q} \asymp \mathbf{d}^9 q^{2\mathbf{d}},$$

as $\mathbf{d} \to \infty$. The summation over Dirichlet characters and their moduli is motivated by Bombieri-Vinogradov Theorem. This result is a function field analogue of [1], but our estimate for the error term in asymptotic formula is sharper than [1].

1. J. B. Conrey, H. Iwaniec, K. Soundararajan, The sixth power moment of Dirichlet L-functions, Geometric and Functional Analysis, 2012, 22, 1257 - 1288.
2. G. Djanković, D. Đokić, The sixth power moment of Dirichlet L-functions over rational function fields. Journal of Mathematical Analysis and Applications, 514(1): 126296, 2022.

Hypersurfaces of the homogeneous nearly Kähler $\mathbb{S}^3 \times \mathbb{S}^3$ with $\mathcal{P}-$slant normal vector field

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In joint work with M. Djoric and M. Moruz, we study the hypersurfaces of $\mathbb{S}^3 \times \mathbb{S}^3$ with a $\mathcal{P}-$slant normal vector field ξ, which satisfy the condition $P\xi = \cos\theta\xi + \sin\theta J\xi$ for some constant angle function θ, where J and P are respectively the almost complex and almost product structures on $\mathbb{S}^3 \times \mathbb{S}^3$. We prove that the only possibilities for θ are $\pi, \frac{\pi}{3}, -\frac{\pi}{3}$ and that all such hypersurfaces are Hopf, with either 3 or 5 different principal curvatures. The main results are the complete classification of the hypersurfaces of $\mathbb{S}^3 \times \mathbb{S}^3$ with $\mathcal{P}-$slant normal and with three different principal curvatures, as well as the classification of such hypersurfaces of $\mathbb{S}^3 \times \mathbb{S}^3$ with five different principal curvatures which are all additionally constant.

Rough homogenization for diffusions on fluctuating Helfrich surfaces

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We will consider lateral diffusion on Helfrich surfaces, possessing both spatial and temporal fluctuations. The covariance of the Brownian particle on a fluctuating Gaussian hypersurface is given by (the ultra-violet cutoff of) the Helfrich energy. Moreover, the generator of the Brownian motion on the surface, in local coordinates, is the Laplace-Beltrami operator. Utilizing the results from homogenization theory, Duncan et al. proved under the assumption of scale separation between the characteristic length and time scales of the surface fluctuations and of the diffusing particle, that the lateral diffusion process can be well approximated by a Brownian motion on the plane with constant diffusion tensor. Diffusion tensor depends in a highly nonlinear way on the properties of the surface. We extend their results proving the convergence towards a particular lift of the homogenization limit in rough paths topology, for which (in certain regimes) a correction term to the Itô iterated integrals appears.

This is a joint work with H. Kremp and N. Perkowski.

Umbilical CR submanifolds of the nearly Kähler $S^3 \times S^3$

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Recently, the investigation of a CR submanifolds of the nearly Kähler manifold $S^3 \times S^3$ was started. After obtained classifications with respect to positions which the distributions D_1, D_2 and D_3 have under an action of the almost product structure P, obtained are some classes of the CR submanifolds of $S^3 \times S^3$ which have an umbilical section. It is proved that CR submanifolds of the nearly Kähler manifold $S^3 \times S^3$ with umbilical sections must have dimension three and then we obtain some examples of them with distinguished vector fields. Also, we classify minimal submanifolds that have a vector field E_4 as an umbilical section. The main result is classification of the three-dimensional umbilical CR submanifolds with totally geodesic an almost complex distribution.

On dynamics of simplest skew products on n-dimensional ($n \geq 2$) cells, cylinders and tori

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We study the structure of the nonwandering set and the topological entropy of some skew products on such n-dimensional ($n \geq 2$) manifolds as cells, cylinders and tori.

LG/CY correspondence between tt^*-geometry

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Let f be a nondegenerate homogeneous polynomial of degree $n+2$ with $n+2$ variables, then it defines a Calabi-Yau hypersurface X in $n+1$-dimensional projective space or a Landau-Ginzburg model with a superpotential f defined on the complex $n+2$-dimensional space, both have tt^*-geometry structure. We proved that there exists a tt^*-substructure on the LG-side, which is isomorphic to the tt^*-structure constructed from the variation of Hodge structures in the Calabi-Yau side. This is a recent joint work with Tian Lan and Zongrui Yang.

Spherical and planar ball bearings as nonholonomic problems

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Nonholonomic systems of n homogeneous balls with the same radius r that are rolling without slipping around a fixed sphere with center O and radius R are constructed. It is assumed that a dynamically nonsymmetric sphere of radius $R+2r$ and the center that coincides with the center O of the fixed sphere rolls without slipping over the moving balls. We prove that these systems possess an invariant measure. In the limit, when the radius R tends to infinity we obtain a corresponding planar problem consisting of n homogeneous balls with the same radius r that are rolling without slipping over a fixed plane, and a moving plane that moves without slipping over the homogeneous balls. We prove that this system possesses an invariant measure and that it is integrable in quadratures according to the Euler-Jacobi theorem. This is joint work with Vladimir Dragović and Božidar Jovanović.

Conformally flat Kähler structures

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It is well-known that any Kähler structure of dimension > 4 is locally conformally flat if and only if it is flat. In the four-dimensional case they are locally symmetric but a complete description was still incomplete in the pseudo-Riemannian setting. The purpose of this lecture is to fill this gap showing a complete description of locally conformally flat Kähler structures. We pay special attention to an interesting connection between Kähler and para-Kähler structures is appearing, as well as to their realization as left-invariant metrics on Lie groups.

On noncommutative branched coverings

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Let X be a compact Hausdorff space and let $C(X)$ be the C^*-algebra of continuous complex-valued functions on X. A $C(X)$-*algebra* is a unital C^*-algebra A endowed with a unital injective $*$-homomorphism from $C(X)$ to the centre of A. The concept of $C(X)$-algebras was introduced by G. Kasparov and may be considered as generalized bundles (or fields) of C^*-algebras, parametrized by the space X.

In 2011, A. Pavlov and E. Troitsky introduced the concept of *noncommutative branched covering*, as a pair (A, B) consisting of a C^*-algebra A and its C^*-subalgebra B with common identity element, such that there exists a conditional expectation of (topologically) finite index from A onto B.

In this talk I shall consider the problem of characterizing those unital $C(X)$-algebras A for which the pair $(A, C(X))$ defines a noncommutative branched covering.

The talk is based on a joint work with Étienne Blanchard.

Multifaceted curl forces and noncentral forces

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Curl force was introduced by Berry and Shukla in 2012. In general, this is not a Hamiltonian force.

After giving a gentle introduction to the Hamiltonian formalism of curl forces we demonstrate their applications in optical trapping, Paul trapping etc, we describe noncentral forces. We will study the reduction of noncentral forces using the Emden-Fowler equation. We describe the first integrals of these equations using Djukic and other methods. We also describe noncentral forces on constant curvature spaces. Finally we end our talk with a geometric description of curl forces.

On rotary mappings

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The lecture is devoted to rotary mappings of two-dimensional space with affine connection onto (pseudo-) Riemannian spaces. We have found the new properties of these mappings, i.e. we found all Riemannian metrics that admit these mappings. An interesting fact is that not all of these metrics are isometric to surfaces of revolution (as it was wrongly stated in the paper of S.G. Leiko). We study the rotary mappings of surfaces of revolution in detail. We have constructed the examples of rotary mappings onto sphere using special projection types.

Classification of tensors on 4-dimensional manifolds of neutral signature

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The algebraic classification of special types of tensors has been used to good effect in many places in differential geometry and physics. Here the situation of a manifold with metric of neutral signature $(+,+,-,-)$ is considered. As would be expected this is more complicated than the positive definite and Lorentz cases but it has been studied in some depth. One important case is when the tensor is a second order, symmetric tensor T and the possible algebraic classes for this have been given directly in the Jordan-Segre notation. An alternative method will be described here where, first, one finds the trace-free part, $T*$, of T, which has the same Jordan-Segre type as T, and considers $T*$ instead. One the links $T*$ uniquely with a 4th order tensor E (with curvature tensor index symmetry) and from which $T*$ can be recovered uniquely. One can then take advantage of the large bank of algebraic knowledge of such 4th order tensors to effect a classification of E and hence of T and this will be described. Some further remarks will be made about general algebraic classification in this signature.

Surfaces immersed in $\mathfrak{so}(n+1)$, related to conformal harmonic maps into S^n, for $n > 2$

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It is a well-known fact that any non-conformal harmonic map $\varphi : \Sigma \to S^2$ from a simply connected Riemann surface Σ into the round 2-sphere $S^2 \subset \mathbb{R}^3$ is the Gauss map of a constant gaussian curvature surface, $F : \Sigma \to \mathbb{R}^3$, and also of two parallel constant mean curvature surfaces, $F^{\pm} = F \pm \varphi : \Sigma \to \mathbb{R}^3$. The surface F integrates the $\mathbb{R}^3$-valued closed 1-form $\eta = \varphi \times (d\varphi \circ J)$, where $\times$ denotes the standard cross product of $\mathbb{R}^3$, and J denotes the complex structure of Σ.
For higher dimensional spheres $S^n, n \geq 3$, we show that the harmonicity of a smooth map $\varphi : \Sigma \to S^n$ amounts to the closedness of the $\mathfrak{so}(n+1)$-valued differential 1-form $\eta = \varphi \bowtie (d\varphi \circ J)$, where now $\tilde{\times}$ is the skew-symmetric bilinear map

$$\bowtie: \mathbb{R}^{n+1} \times \mathbb{R}^{n+1} \to \mathfrak{so}(n+1), \quad x \bowtie y = x.y^t - y.x^t$$

This means that we can integrate on simply-connected domains Σ in order to obtain a map $F : \Sigma \to \mathfrak{so}(n+1)$. Thus if φ is a conformal harmonic immersion, then F is a conformal immersion which, in contrast with the 3-dimensional case $F : \Sigma \to \mathbb{R}^3 \equiv \mathfrak{so}(3)$, (where F is necessarily a totally umbilical surface) F can exhibit a variety of interesting geometrical behaviors which we shall describe. Also we give some remarks for higher dimensional complex domains Σ.

Cross graphs: checkerboard embeddings, intersection graphs and minor theory

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We consider cross graphs, i.e. 4-valence graphs with the structure of opposite edges, and their checkerboard embeddings in 2-surfaces. A smoothing at a vertex can be defined for cross graphs. It is not hard to see that a cross graph obtained from an embedded cross graph by smoothings (and possibly by deleting some connected components) can also be embedded into the same surface. Therefore, we can define a minor of a cross graph and the property of a graph to be embedded in a given surface is minor-closed. The aim is to find a criterion for checkerboard embedding into a surface in terms of excluded minors. Manturov's criterion for checkerboard embedding of a cross graph is formulated in terms of ranks of induced subgraphs of a simple graph constructed by using an Euler tour on the cross graph. We investigate this property for excluded minors and also find lists of excluded minors for embedding in some surfaces.

Solution of the qc Yamabe equation on a 3-Sasakian manifold, extremals of the Sobolev-Folland-Stein inequality on the quaternionic Heisenberg groups and the qc Yamabe problem

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A complete solution to the quaternionic contact Yamabe equation on the $(4n + 3)$-dimensional sphere as well as on the quaternionic Heisenberg group is given and a uniqueness result for a compact locally 3-Sasakian manifold is shown. Consequently, the best constant and all extremals of the Sobolev-Folland-Stein inequality on the quaternionic Heisenberg group are determined. The quaternionic contact Yamabe problem is solved on compact non-locally spherical quaternionic contact manifolds.

Geometry of Bier spheres

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Bier sphere $Bier(K) = K *_\Delta K^\circ$ is a simplicial complex defined as join of simplicial complex K and its Alexander dual. It is a combinatorial sphere with no a priori geometric realisation, and it is knows that most of Bier spheres are not polytoplal. We study the geometry of Bier spheres by describing their natural geometric realisation as star-shaped bodies, and provide effective criteria for their polytopality which clarifies the connection between Bier spheres and generalised permutohedra. We also establish a connection of Bier spheres of maximal volume with generalisation of the classical Van Kampen-Flores theorem. (Joint work with Rade Živaljević.)

Bogoyavlenski conjecture and classification of multiplicity free and almost multiplicity free subgroups

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In [Bogoyavlenski, O.I.: *Integrable Euler equations associated with filtrations of Lie algebras*, Mat. Sb. **121(163)** (1983) 233–242] is conjectured that if the Euler equations on a Lie algebra $\mathfrak{g}_0$ are integrable, then their certain extensions to semisimple lie algebras $\mathfrak{g}$ related to the filtrations of Lie algebras

$$\mathfrak{g}_0 \subset \mathfrak{g}_1 \subset \mathfrak{g}_2 \cdots \subset \mathfrak{g}_{n-1} \subset \mathfrak{g}_n = \mathfrak{g}$$

are integrable as well.

In particular, by taking $\mathfrak{g}_0 = \{0\}$ and natural filtrations of $so(n)$ and $u(n)$, we have Gel'fand-Cetlin integrable systems. We proved the conjecture for filtrations of compact Lie algebras $\mathfrak{g}$: the system are integrable in a noncommutative sense by means of polynomial integrals. Various constructions of complete commutative polynomial integrals for the system are also given.

In addition, related to commutative polynomial integrability, we classify almost multiplicity free subgroups of compact simple Lie groups, see [Guillemin, V and Sternberg, S.: *Multiplicity-free spaces*, J. Diff. Geometry **19** (1984) 31–56], [Krämer, M.: Multiplicity free subgroups of compact connected Lie groups. Arch. Math. **27** (1976) 28–36].

Knot dynamics

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We examine computer experiments that can be performed to understand the dynamics of knots under self-repulsion. We find unknotted configurations that will not reduce to simple circular forms under self-repulsion, and we find complex versions of knots that will not reduce to simpler forms under the self-repulsion. The phenomena that we have discovered depend very little on the details of the computer program as long as it conforms to a general description of self-repulsion. Thus, we suggest on the basis of our experiments that sufficiently complex examples of hard unknots and sufficiently complex examples of complexified knots will not reduce to global minimal energy states in self-repulsion environments. It is a challenge to other program environments to verify or disprove these assertions.

Some properties of the quasi-conformal diffeomorphisms of the unit disc

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We will give a new idea how to consider some obtained distortion results of the class of HQC diffeomorphisms of the unit disc $\mathbb{D}$ in order to get new Schwarz-Pick type results for that class of functions. In particular, we will give the answers to many questions concerning those mappings which are related to the determination of different properties that are of essential importance for validity of the results such as those that generalize famous inequalities of the Schwarz-Pick type. The approach used is geometrical in nature.

Real hypersurfaces in $S^6(1)$ equipped with structure Jacobi operator satisfying $\mathcal{L}_X l = \nabla_X l$

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It is well known that the sphere $S^6(1)$ admits an almost complex structure J which is nearly Kähler. If M is a hypersurface of an almost Hermitian manifold with a unit normal vector field N, the tangent vector field $\xi = -JN$ is said to be characteristic. The Jacobi operator with respect to ξ is called structure Jacobi operator and is denoted by $l = R(\cdot, \xi)\xi$, where R is the curvature tensor on M.

We investigate real hypersurfaces in nearly Kähler sphere $S^6(1)$ whose Lie derivative of structure Jacobi operator coincides with the covariant derivative of it and show that such submanifolds do not exist.

Symplectic invariants of structurally stable singularities of integrable systems

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We study singularities of the Lagrangian fibration given by a completely integrable system. We prove that a compact non-degenerate singular fibre satisfying the so-called connectedness condition is structurally stable under (small enough) real-analytic integrable perturbations of the system. In other words, the topology of the fibration in a neighbourhood of such a fibre is preserved after any such perturbation. We also give a classification, up to real-analytic symplectic equivalence, of the Lagrangian fibrations in a neighbourhood of such a fibre.

Pell and Pell-Lucas hybrid quaternions

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In this study, we work on new identities about the Pell and Pell-Lucas numbers and the Pell and Pell-Lucas quaternions. Firstly, the Pell and Pell-Lucas hybrid quaternions are introduced, then using Pell and Pell-Lucas numbers and quaternions, identities and properties of Pell and Pell hybrid quaternions, such as the Cassini's identity and the Binet's formula, are given.

Braidings, braid equivalences and Jones-type invariants

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We will present algorithms for turning knots and links into braids in various diagrammatic settings and the corresponding braid equivalences up to link isotopy. Then we will explain the construction of Jones-type knot and link invariants via Markov traces on appropriate quotient algebras of braid groups.

Geodesic equations in the weak field limit of general $f(R)$ gravity theory

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In our work we presented the modified field equations generated by action with unspecified function $f(R)$. Assuming spherical symmetry, we derived the corresponding static Schwarzschild-like metric in the weak field limit. Also we considered geodesic equations of motion describing orbits and orbital speeds which can be measured in galactic environment. We solved geodesic equations in the case of a power-low potential of the form $(r/r_c)^\beta$, where r_c is a characteristic length scale.

Generalized virtual polytopes and cohomology rings of torus manifolds

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In 1992 Pukhlikov and Khovanskii obtained a description of an intersection ring of a complex nonsingular projective toric variety via a volume polynomial of a virtual polytope.A topological generalization of a complex nonsingular projective toric variety was introduced and studied by Davis and Januszkiewicz in 1991 and is known in toric topology as a (quasi)toric manifold. They showed that cohomology rings of quasitoric manifolds are isomorphic to quotient rings of Stanley-Reisner algebras of simple polytopes by linear ideals. Quasitoric manifolds form an important family of torus manifolds, defined by Masuda in 1999.In this talk, I'm going to introduce a theory of volume polynomials of generalized virtual polytopes based on the study of topology of affine subspace arrangements in a real Euclidean space. Then I will show how to apply this theory in order to obtain a topological version of the classical BKK Theorem, the Stanley-Reisner and Pukhlikov-Khovanskii type descriptions for cohomology rings of a wide class of torus manifolds, which we call generalized quasitoric manifolds. The talk is based on a joint work with Askold Khovanskii (University of Toronto) and Leonid Monin (Max Planck Institute for Mathematics in the Sciences).

On quiver representations of digraphs associated with finite rings

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In the present talk we shall report on the ongoing research on connection between digraphs associated to finite (commutative) rings and quiver representations.

Digraph associated to a finite ring A has the set of vertices $V = A^2$ and arrows (or edges) $E = \{(a, b) \to (a+b, ab)\}$. In another terminology, it is a finite quiver with loops. In addition to our previous work ([1-3]) to understand the graph itself and especially the loops and their connection to the original ring, the main goal of the present work is to analyze this quiver from the point of view of quiver representations.

Literature:

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[2] А. Липковски: Структурные графы колец: определения и первые результаты. Фундамент. и прикл. матем., 20:3, 2015, 181–190 (A. T. Lipkovski, "Structure graphs of rings: definitions and first results", Fundam. Prikl. Mat., 20:3, 2015, 181–190).

[3] J. Škorić, A. Lipkovski: Finite rings and their structural graphs. International Congress on Mathematics MICOM-2015, Athens, 22.-24. september 2015.

The Ahlfors-Schwarz lemma, curvature, distance and distortion

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We consider versions of Ahlfors-Schwarz lemma related to ultrahyperbolic metric, quasiconformal harmonic and holomorphic functions (including several dimensions versions) and harmonic mappings. Our exposition includes some recently obtained results.

Gaussian curvature conjecture for minimal graphs

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We solve the longstanding Gaussian curvature conjecture of a minimal graph S over the unit disk. This conjecture states the following : For any minimal graph lying above the entire unit disk, the Gauss curvature at the point above the origin satisfies the sharp inequality $|\mathcal{K}| < \pi^2/2$. The conjecture is first reduced to the estimation of the Gaussian curvature of certain Scherk type minimal surfaces over some bicentric quadrilaterals inscribed in the unit disk containing the origin. Then we make a sharp estimate of the Gaussian curvature of those minimal surfaces over those bicentric quadrilaterals at the point above the zero. The talk is based on the joint paper with David Kalaj.

On geodesic mappings and their generalizations

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The lecture will be devoted to certain problems of the theory of geodesic mappings. We will present the fundamental equations of these mappings. We will consider geodesic mappings of some special spaces. We will also point out the connection to the rotary mappings.

Geodesic mappings of generalized Einstein spaces onto Riemannian space

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The paper deals with Einstein type tensors and some relations that they satisfy in the generalized Riemannian space. Also, we consider geodesic mappings of generalized Einstein spaces onto Riemannian space.

On almost geodesic mappings of the second type of spaces with non-symmetric affine connection and Nijenhuis tensor

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The talk deals with some new form of the basic equations of almost geodesic mappings of the second type of spaces with non-symmetric affine connection. The main results of the work are necessary and sufficient conditions that a space with non-symmetric affine connection permits an almost geodesic mapping. These results were found using the Nijenhuis tensor.

Einstein derivations of nilpotent Lie algebras

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Consider a nilpotent Lie group G with a left invariant Riemannian metric g. The metric g defines the Euclidean inner product on the Lie algebra $\mathfrak{g}$ of G. The left-invariant Ricci tensor Ric of the metric g defines a self-adjoint Ricci operator $R : \mathfrak{g} \to \mathfrak{g}$. Fixing some orthonormal basis $e_1, \dots, e_n$ in the metric *nilpotent* Lie algebra $\mathfrak{g}$, one can write R as

$$R = \frac{1}{4}\sum_{i=1}^{n} ad_{e_i} ad^*_{e_i} - \frac{1}{2}\sum_{i=1}^{n} ad^*_{e_i} ad_{e_i},$$

where $ad^*_{e_i}$ denotes the adjoint operator to $ad_{e_i}, ad_{e_i}(x) = [e_i, x]$.

The operator R, like the identity operator Id, is not a derivation of the Lie algebra $\mathfrak{g}$, but it may happen that for some constant $c \in \mathbb{R}$ the operator $D = R - cId$ will be a derivation of the Lie algebra $\mathfrak{g}$ and in this case the derivation D is called the *Einstein derivation* of the Lie algebra $\mathfrak{g}$ [1]. To search for Einstein derivations for "almost-graded" Lie algebras, Nikolaevsky proposed the notion of pre-Einstein derivation.

I will try to talk about geometric problems that lead to the search for Einstein differentiations, as well as about their connection with *positive gradings* of Lie algebras.

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Canonical Weierstrass representation of minimal Lorentz surfaces in pseudo-Euclidean 4-space with neutral metric

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The minimal Lorentz surfaces in the pseudo-Euclidean 4-space with neutral metric $\mathbb{E}_2^4$ whose first normal space is two-dimensional and whose Gauss curvature K and normal curvature $\varkappa$ satisfy the condition $K^2-\varkappa^2\neq 0$ are called minimal Lorentz surfaces of general type. We prove that these surfaces admit special isothermal parameters, called canonical. The Gauss curvature K and the normal curvature $\varkappa$ of such a surface considered as functions of the canonical parameters satisfy the following system of natural PDEs:

$$\begin{aligned} \sqrt[4]{\left|K^2-\varkappa^2\right|}\,\Delta^h \ln\left|K^2-\varkappa^2\right| &= 8K; \\ \sqrt[4]{\left|K^2-\varkappa^2\right|}\,\Delta^h \ln\left|\tfrac{K+\varkappa}{K-\varkappa}\right| &= 4\varkappa; \end{aligned} \qquad K^2-\varkappa^2\neq 0. \quad (1)$$

We find a Weierstrass representation with respect to isothermal parameters of any minimal Lorentz surface of general type. Further, we obtain a Weierstrass representation with respect to canonical parameters and describe all these surfaces in terms of four real functions. Using the canonical Weierstrass representation we solve explicitly system (1) expressing any solution to this system by means of four real functions of one variable. Finally, by means of the canonical Weierstrass representation formula we give examples of minimal Lorentz surfaces of general type in $\mathbb{E}_2^4$ parametrized by canonical parameters.

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Convex functions and isometric actions on Lorentzian manifolds

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We give some theorems and examples to show that there are strong connections between the existence of convex functions on Riemannian and Lorentzian manifolds and topological properties of the orbits and orbit spaces arising from isometric actions.

Hyperbolic crystal geometry
To honour of János Bolyai on the 220th anniversary of his birth

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In analogy of the 230 Euclidean crystallographic space groups we describe finitely many infinite series (by some integer parameters) of hyperbolic space groups where the fundamental domains are simplices and truncated simplices or their integer parts.

The principle of classification is characterized by the so called D-symbol method (in honour of M.S. DELANEY, B.N. DELONE (DELAUNAY) and A.W.M. DRESS) in former works of the authors and in the recent comprehensive paper, to appear in journal Filomat (Niš University).

We mention and illustrate with figures also actual (solved but merely unsolved) extremal problems, as densest ball packings, loosest ball coverings and their optimal arrangements, which are timely also in modern material sciences.

Thus, the discovery of János BOLYAI and Nikolai I. LOBACHEVSKY, hyperbolic geometry, gets new and new importances and perspectives. Nowadays also other homogeneous THURSTON geometries come into account.

Magnetic Jacobi fields in almost contact metric manifolds

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The first variation of the Landau Hall functional on a Riemannian manifold leads to the notion of magnetic curves. Computing the second variation, we obtain the equation of a Jacobi-type field along a magnetic curve. In this talk we focus on the contact magnetic trajectories in Sasakian and cosymplectic manifolds, respectively (as ambient space) emphasising the main differences between the two cases. We will give several examples.

Small deformations, variations and energies of curves

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Contrary to isometries as transformations that preserve distances, infinitesimal bending is a type of small deformation in which the arc length is invariant with appropriate precision. The influence that given deformations have on geometric quantities is measured by variations. In this talk we discuss the infinitesimal bending of curves and variations of their energies. In particular, we consider spherical curves and an infinitesimal bending field that leaves a given curve on a sphere with a predetermined deviation.

This is a joint work with Ljubica Velimirović, Svetozar Rančić and Miroslav Maksimović.

Generalized Darboux frame of a spacelike curve lying on a lightlike surface in Minkowski 3-space

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This is a joint work with Jelena Djordjević (University of Kragujevac, Faculty of Science, email: jelenadjordjevic23@yahoo.com)

We show that there is a more general way to frame a spacelike curve α with non-null principal normal lying on a lightlike surface in Minkowski space $\mathbb{E}_1^3$, by introducing a generalized Darboux frame of such a curve. We prove that α has two possible generalized Darboux frames and give geometric interpretations of their vector fields. We define geodesic curvature, normal curvature, and geodesic torsion according to the generalized Darboux frame and obtain the generalized Darboux frame's equations. We also show that there are no spacelike asymptotic curves according to the generalized Darboux frame and find the necessary and sufficient condition for a spacelike curve lying on a lightlike surface to be a geodesic curve and principal curvature line according to the generalized Darboux frame. In particular, we investigate when the generalized Darboux frame and Darboux frame of α have the same axis of rotation. Finally, we provide some examples.

Finite homogeneous metric spaces with special properties

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This talk is devoted to the study of finite homogeneous metric spaces with special properties. Vertex sets of polytopes in Euclidean spaces are considered as finite metric spaces usually in this talk. The main part is based on recent papers written in collaboration with Professor V. N. Berestovskii.

Let us recall that a metric space (M, d) is called homogeneous if for every points $x, y \in M$, there is a self-isometry f of the space (M, d) such that $f(x) = y$. There are several properties of metric spaces, that can be considered as a strengthening of the homogeneity property. These properties include the (generalized) normal homogeneity, Clifford – Wolf homogeneity, two-point homogeneity, n-point homogeneity, $n \geq 2$.

It is clear that a finite metric subspace of a Euclidean space that is "strengthened" homogeneous must be "more symmetric" in some sense. Another type of the symmetry in Euclidean space is provided by the vertex sets of regular and semiregular polytopes and their natural generalizations.

In this talk, we are going to discuss numerous connections between different concepts of the symmetry. In particular, we will discuss the classifications of generalized normal homogeneous and Clifford — Wolf homogeneous metric spaces that are the vertex sets of regular or semiregular polytopes in Euclidean spaces.

Manifold hypothesis in data analysis. Dimension estimation.

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Manifold hypothesis states that data points (especially obtained by a neural network) in high-dimensional space actually lie in a vicinity of a much lower-dimensional manifold. Manifold hypothesis got certain empirical verification. It is rather important in the area of unsupervised and semi-supervised learning. The talk is devoted to the problem of dimension estimation of such manifolds. New method is suggested and tested in a certain case of face-recognition neural network.

This research was supported by Huawei and conducted in collaboration with A. Chekunov, D. Fedoseev, A. Ivanov, V. Kibkalo, S. Komkov, I. Mazurenko, M. Nikulin, A. Petyushko, F. Popelenskiy.

Holomorphic foliations on complex moment-angle manifolds

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Moment-angle manifolds provide a wide class of examples of non-Kaehler compact complex manifolds with a holomorphic torus action. A complex moment-angle manifold Z is constructed via a certain combinatorial data, called a complete simplicial fan. In the case of rational fans, the manifold Z is the total space of a holomorphic bundle over a toric variety with fibres compact complex tori. In this case, the invariants of the complex structure of Z, such Dolbeault cohomology and the Hodge numbers, can be analysed using the Borel spectral sequence of the holomorphic bundle.

In general, a complex moment-angle manifold Z is equipped with a canonical holomorphic foliation F which is equivariant with respect to the algebraic torus action. Examples of moment-angle manifolds include the Hopf manifolds, Calabi–Eckmann manifolds, and their deformations. The holomorphic foliated manifold (Z,F has been also studied as a model for irrational ("non-commutative") toric varieties in the works several authors (arXiv:1308.2774, arXiv:2002.03876).

We construct transversely Kaehler metrics on moment-angle manifolds Z, under some restriction on the combinatorial data. We prove that all Kaehler submanifolds in such a moment-angle manifold lie in a compact complex torus contained in a fibre of the foliation F. For a generic moment-angle manifold Z in its combinatorial class, we prove that all its subvarieties are moment-angle manifolds of smaller dimension. This implies, in particular, that Z does not have non-constant meromorphic functions, i.e. its algebraic dimension is zero.

Battaglia and Zaffran (arXiv:1108.1637) computed the basic Betti numbers for the canonical holomorphic foliation on a moment-angle manifold Z corresponding to a shellable fan. They conjectured that the basic cohomology ring in the case of any complete simplicial fan has a description similar to the cohomology ring of a complete simplicial toric variety due to Danilov and Jurkiewicz. We prove the conjecture. The proof uses an Eilenberg-Moore spectral sequence argument; the key ingredient is the formality of the Cartan model for the torus action on Z.

The talk is based on joint works with Hiroaki Ishida, Roman Krutowski, Yuri Ustinovsky and Misha Verbitsky.

On generalized almost para-Hermitian spaces

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Very recently, generalized almost Hermitian spaces were defined in (Petrović, M.Z., Velimirović, L.S. Generalized Almost Hermitian Spaces and Holomorphically Projective Mappings. Mediterr. J. Math. 17, 74 (2020)).

In the same manner we can study generalized almost para-Hermitian spaces. Some properties of these spaces and special generalized almost para-Hermitian spaces including generalized para-Hermitian spaces as well as generalized nearly para-Kähler spaces are discussed.

Finally, an example of a generalized para-Kähler space in Eisenhart's sense is given.

Boundary-sensitive Hodge decompositions

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We provide a theoretical framework for discrete Hodge-type decomposition theorems of piecewise constant vector fields on simplicial surfaces with boundary that is structurally consistent with decomposition results for differential forms on smooth manifolds with boundary. In particular, we obtain a discrete Hodge-Morrey-Friedrichs decomposition with subspaces of discrete harmonic Neumann fields $H_{h,N}$ and Dirichlet fields $H_{h,D}$, which are representatives of absolute and relative cohomology and therefore directly linked to the underlying topology of the surface. In addition, we discretize a recent result that provides a further refinement of the spaces $H_{h,N}$ and $H_{h,D}$, and answer the question in which case one can hope for a complete orthogonal decomposition involving both spaces at the same time.

Exciting open questions are related to the so-called Poincaré-angle which appears as a feature of the decomposition on surfaces with positive genus. As applications, we present a simple strategy based on iterated L^2-projections to compute refined Hodge-type decompositions of vector fields on surfaces, which gives a more detailed insight than previous decompositions. As a proof of concept, we explicitly compute harmonic basis fields for the various significant subspaces and provide exemplary decompositions for two synthetic vector fields. All techniques are essential for vector field analysis, surface parametrization, remeshing and others. We will show several applications.

On bi-Hermitian surfaces

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We call bi-Hermitian surface, a smooth manifold of complex dimension two endowed with a Riemannian metric g which is Hermitian with respect to two complex structures J_1 and J_2 which are different ($J_1 \neq \pm J_2$) but induce the same orientation. We report on what is known about the existence and non- existence of such structures in the compact case.

Combinatorial Ricci flows on closed surfaces

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It was shown that the Ricci flow on a closed surface converges to a metric of constant curvature for any initial metric (in contrast with 3d case). There are at least two important versions of discrete Ricci flow for 2d case. We discuss results obtained for discrete versions of the Ricci flow on triangulated closed surfaces.

Billiard games and books

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The aim of this work is to put together two novel concepts from the theory of integrable billiards: billiard ordered games and confocal billiard books. Billiard books appeared recently in the work of Fomenko's school, in particular of V. Vedyushkina. These more complex billiard domains are obtained by gluing planar sets bounded by arcs of confocal conics along common edges. Such domains are used in this paper to construct the configuration space for billiard ordered games. We analyse dynamical and topological properties of the systems obtained in that way.

On the geometry of a weakened f-structure

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An f-structure, introduced by K.Yano in 1963 and subsequently studied by a number of geometers, is a higher dimensional analog of almost complex and almost contact structures, defined by a (1,1)-tensor field f on a $(2n+p)$-dimensional manifold, which satisfies $f^3+f=0$ and has constant rank $2n$. In this case, the tangent bundle TM splits into two complementary subbundles: a $2n$-dimensional $f(TM)$ and a p-dimensional $\ker f$;moreover, the restriction of f to $f(TM)$ determines a complex structure.We recently introduced [1,2] the weakened f-structure (i.e., the complex structure on $f(TM)$ is replaced by a nonsingular skew-symmetric tensor)and its subclasses of weak K-, S-, and C- structures on Riemannian manifolds with totally geodesic foliations, which allow us to take a fresh look at the classical theory. We demonstrate this by generalizing several known results on globally framed f-manifolds.First, we express the covariant derivative of f using a new tensor on a weak metric f-structure, then we prove that on a weak K-manifold the characteristic vector fields are Killing and $\ker f$ defines a totally geodesic foliation, an S-structure is rigid, i.e., our weak S-structure is an S-structure, and a weak f-structure with parallel tensor f reduces to a weak C-structure [4].For $p=1$ we obtain the corresponding corollaries for weak almost contact, weak cosymplectic, and weak Sasakian structures, see [3].

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Invariant measures of Hamiltonian flows and random walks in Hilbert space

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Finitely additive analogues of Lebesgue measure on a separable Hilbert space are considered. That measures are used for Koopmans representation of Hamiltonian flows and for averaging of walks along a random Hamiltonian fields.

A split special Lagrangian calibration associated with helicity

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Let M be an oriented three dimensional Riemannian manifold. We define a notion of helicity of local sections of the bundle $SO(M) \to M$ of all its positively oriented orthonormal tangent frames. When M is a space form, we relate the concept to a suitable invariant split pseudo-Riemannian metric on $\mathrm{Iso}_o(M) \cong SO(M)$: A local section has positive helicity if and only if it determines a space-like submanifold. In the Euclidean case we find explicit homologically volume maximizing sections using a split special Lagrangian calibration. We introduce the concept of optimal helicity and give an optimal screwed global section for the three-sphere. We prove that it is also homologically volume maximizing (now using a common one-point split calibration). Besides, we show that no optimal section can exist in the Euclidean and hyperbolic cases.

The heat content functional and isoparametric foliations

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We derive the first variation formula for the heat content of a smooth bounded domain in an analytic Riemannian manifold, and see how stationary points force the existence of an isoparametric foliation on the domain. The purpose of the talk is to show the role of isoparametric foliations in the theory of overdetermined PDE's; in many respects they generalize the properties of the standard foliation of Euclidean space by concentric spheres.

On the homotopy colimit description for quotients of moment-angle complexes

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In this talk for any simplicial complex K on m vertices and any closed subgroup H in the compact torus T^m we define a diagram $Q = Q(K, H)$ of tori homomorphisms (a toric diagram) over the category *cat* K. The homotopy colimit of Q is shown to be homotopy equivalent to the quotient Z_K/H of the moment-angle complex Z_K by H with respect to the standard action of T^m on Z_K. A similar result holds for the quotient of the real moment-angle manifold R_K/H by any subgroup H of the real torus $(\mathbb{Z}/2\mathbb{Z})^m$. These results generalize the previously known homotopy colimit descriptions of real and complex moment-angle complexes (Welker-Ziegler-Zivaljevic, Panov-Ray, Franz). We deduce formality of the Borel construction and collapse of theEilenberg-Moore spectral sequencefor a new class of quotients by subgroups in T^m (wider than just free torus actions).

We obtain some results on torsion in the cohomology of partial quotients based on the work of Li Yu (his class is included into our new class of torus actions).

This is a joint project with I.Y. Limonchenko (arXiv:2202.13899).

Nonlocal de Sitter gravity and its exact cosmological solutions

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In the Einstein-Hilbert action, we introduce nonlocality by the following way: $R-2\Lambda = \sqrt{R-2\Lambda}\,\sqrt{R-2\Lambda} \to \sqrt{R-2\Lambda}\,F(\Box)\,\sqrt{R-2\Lambda}$, where $F(\Box) = 1 + \sum_{n=1}^{+\infty}\left(f_n\Box^n + f_{-n}\Box^{-n}\right)$ is an analytic function of the d'Alembert-Beltrami operator $\Box$ and its inverse $\Box^{-1}$. Derivation of equations of motion for gravitational field $g_{\mu\nu}$ is presented in [2]. To solve the corresponding equations of motion, we first solve equation $\Box\sqrt{R-2\Lambda} = q\sqrt{R-2\Lambda}$, where $q = \zeta\Lambda \quad (\zeta \in \mathbb{R})$ is an eigenvalue and $\sqrt{R-2\Lambda}$ is an eigenfunction of operator $\Box$. We presented several exact cosmological solutions for homogeneous and isotropic universe. One of these solutions mimics effects that are usually assigned to dark matter and dark energy, see [1]. Some other solutions are examples of the nonsingular bounce ones in flat, closed and open universe. There are also singular and cyclic solutions. All these cosmological solutions are a result of nonlocality and do not exist in the local de Sitter theory of gravity.

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Geometry in the large on Hadamard manifolds

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In our talk, we prove several Liouville-type theorems on the non-existence of Killing-Yano, Killing and symmetric harmonic tensors on Hadamard manifolds and, in particular, on Riemannian symmetric spaces of non-compact type. These theorems supplement the well-known vanishing theorems for the above tensors, obtained using the Bochner technique for compact Riemannian manifolds. In turn, the proofs of our theorems will use well-known Liouville-type theorems on the non-existence of subharmonic and harmonic functions on complete Riemannian manifolds, which we have partially modified for the case of Hadamard manifolds.

Eigenproblem of tensors - a geometrical viewpoint

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Classical eigenproblem considers eigenvalues and eigenvectors of linear operator within vector space. Matrix representation of the problem has been extended towards multidimensional arrays that have various applications. Another extension of the problem has been directed to invariant subspaces of multilinear operators in Banach spaces. Generalization of eigenproblem for tensors is challenging. Eigenproblem of supersymmetric tensors on Riemannian manifold will be considered. Proper definition will be given and compared with the extensions. Examples will be pointed.

New invariants of knots and 3-manifolds

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In this talk I will give an overview of some recent results on invariants of knots, and of 3-manifolds. This will be also related with the knots-quivers correspondence, that was originally introduced by rewriting colored HOMFLY-PT invariants in the form of quiver generating series for suitable quivers. Recently, other knot invariants have also been shown to be re-writable in a "quiver form", namely Gukov–Manolescu $\hat{Z}$-invariants of knot complements. I will present some basic facts about these correspondences, as well as relations between them.

On the Roter type of generalised Wintgen ideal Legendrian submanifolds

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I. Mihai obtained an inequality relating intrinsic normalised scalar curvature and extrinsic squared mean curvature and normalised normal curvature of Legendrian submanifolds M^n in Sasakian space forms $M^{2n+1}(c)$. In this paper, for the class of generalised Wintgen ideal submanifolds M^n of Sasakian space form $\widetilde{M}^{2n+1}(c)$, we study relationship between some properties concerning their Deszcz symmetry and their Roter type.

Dynamics of convex mean curvature flows

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We know many convex ancient solutions to MCF. Instead of considering these ones at a time, one can consider the space of all convex hypersurfaces and regard rescaled mean curvature flow as a flow on this space. Then one can ask what the fixed points are, what connecting orbits between fixed points exist, and when new ancient solutions are found one can ask where they fit in this picture. Besides these things we will also discuss some topological properties of that space, such as path conectedness, compactness property etc. This is a joint work with Angenent and Daskalopoulos.

Geometry of cotangent space of Heisenberg group

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The cotangent bundle of Heisenberg group is 6-dimensional, 2-step nilpotent group with 3-dimensional center. In this work we classify, up to automorphisms, left-invariant pseudo Riemannian metrics on the cotangent bundle of Heisenberg group.

Particularly interesting is case of metrics with Lorentzian central restriction. The classification of such metrics is directly related to the classification of conics in the hyperbolic plane.

On the formality problem for manifolds with special holonomy

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We discuss the formality problem for manifolds with special holonomy. As an example we consider a certain G_2 manifold constructed by Joyce for which we compute the cohomology ring by using the intersection theory and establish its formality (the latter is a joint result with M. Amann).

Singularities of the T^n-action on the Grassmann manifolds $G_{n,2}$ and their resolution

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We disccus some of the results from [4] on singularities of the orbit space $G_{n.2}/T^n$. The complex Grassmann manifolds $G_{n,2}$ of complex 2-planes in $\mathbb{C}^n$ can be endowed with the canonical action of the compact torus T^n and the standard moment map $\mu : G_{n,2} \to \mathbb{R}^n$, which is defined in terms of Plücker coordinates, which is T^n invariant and its image is the hypersimplex $\Delta_{n,2}$.

The critical points of the moment map μ as defined in mathematical analysis we proved in [2] to coincide with those points in $G_{n,2}$ which have a non-trivial stabilizer in $T^{n-1} = T^n/\mathrm{diag}(T^n)$. Therefore, the notion of a critical point can be extended to the orbit space $G_{n,2}/T^n$.

On the other hand, for a genearal $(2n, k)$-manifold we introduced in [3] the notion of an universal space of parameters $\mathcal{F}$. In [5] we

provided an explicit construction of this space $\mathcal{F}_n$ for the Grassmannians $G_{n,2}$. Moreover, we proved in [4] that there exists the continious projection $p_n : U_n = \mathcal{F}_n \times \Delta_{n,2} \to G_{n,2}/T^n$, that is the orbit space $G_{n,2}/T^n$ is the quotient space of U_n by the relation defined by the map p_n. The space U_n is a manifold with singular corners, while the space $G_{n,2}/T^n$ is not generally a topological manifold as the case $n = 5$ demonstrates [2], or even when it is a topological manifold its induced structure from $G_{n,2}$ may have cone-like singularitites as the case $n = 4$ shows [1].

We approach the notion of singularity from the following point of view. The definition of the universal space of parameters implies that to any Plücker stratum $W_\sigma \subset G_{n,2}$ it can be assigned the virtual space of parameters $\tilde{F}_\sigma \subset \mathcal{F}_n$ and the projection $p_\sigma : \tilde{F}_\sigma \to F_\sigma$, where $F_\sigma = W_\sigma/(\mathbb{C}^*)^n$. The space U_n can be considered as a singular fiber space over $G_{n,2}/T^n$, whose fiber over W_σ/T^n is given by the kernel of the projection $p_\sigma : \tilde{F}_\sigma \to F_\sigma$. In this context, we introduce the notion of a singular point in $G_{n,2}/T^n$ as a point of a stratum whose fiber is non trivial. Thus, we show that the space U_n resolves the singularities on $G_{n,2}/T^n$ including the critical points, since we prove that all critical points are singular as well for $n \geq 5$. In addition, we describe the preimages in U_n of the singular points.

The talk is based on the results jointly obtained with Victor M. Buchstaber.

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4. V. M. Buchstaber and S. Terzić, *A resolution of singularities for the orbit spaces $G_{n,2}/T^n$*, to appear in Proceedings of the Steklov Institute of Mathematics
5. V. M. Buchstaber and S. Terzić, *The orbit spaces $G_{n,2}/T^n$ and the Chow quotients $G_{n,2}//(\mathbb{C}^*)^n$ of the Grassmann manifolds $G_{n,2}$*, arXiv:2104.08858 (2021).

Kähler hyperbolicity and Lang conjecture

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Serge Lang made a conjecture that if X is a projective (or compact Kähler) manifold, then X is of general typer together with all its (even singular) subvaraities.

A similar conjecture by Green and Griffith states that if X is a compact complex manifold of general type, then there exists a proper subvaraity Y (different from X) such that any non constant holomorphic map from the complex line C to X has image contained in Y.

The proof in general of both conjecture seems at the moment out of reach.

In a joint paper with Simone Diverio, Francesco Bei and Philippe Eyssidieux we give proof of both conjecture in the special case of the so called Kähler hyperbolic manifold, a concept introduced by Michel Gromov in 1991.

Global scheme of the basic interactions and their geometrical interpretations

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Our space-time consists of three 3-dimensional spaces: space S which is homeomorphic to S^3, space rotations SR which is also homeomorphic to S^3, and time T which is homeomorphic to $\mathbb{R}^3$. First are considered the basic possible 4 cases for exchange from one space into another: 1. $r \to s$, 2. $s \to r$, 3. $r \to t$, and 4. $s \to t$, where $s \in S$, $r \in SR$, and $t \in T$. The first two exchanges are explained as mechanical effects. The third and the fourth cases lead to the known speed of time in gravitational field, and also to the displacement in time. Further 4 generalized exchanges 1^*, 2^*, 3^*, and 4^*, induced by the cases 1-4 are considered. The cases 1^* and 2^* together lead to strong interaction, weak interaction and the galactic acceleration which is observed at the periphery of the galaxies, the case 3^* leads to magnetic fields of the spinning bodies and electromagnetic interaction, and case 4^* leads to gravitational interaction. It leads to global scheme of the basic interactions and their geometrical interpretations.

Isometric embeddings into Gromov-Hausdorff class

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We deal with the proper class of all metric spaces considered up to isometry and equipped with the famous Gromov-Hausdorff distance. We develop the theory of generic metric spaces X and show that the corresponding metric vector has a neighborhood U in the ambient vector space with sup-norm such that U can be isometrically embedded into the Gromov-Hausdorff class. Moreover, this U can be chosen arbitrarily large by an appropriate choice of X. This leads to an isometric Kuratovski embedding of a given bounded metric space of the same cardinality as X into U and thus into the Gromov-Hausdorff class. This work was done jointly with Ivanov and develops previous results of Ivanov, Stavros Iliadis and the author.

Lagrangian submanifolds of the complex quadric

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The complex quadric Q^n is a complex hypersurface of complex $(n+1)$-dimensional projective space. This manifold inherits a Kähler structure from the complex projective space, carries a family of non-integrable almost product structures and its curvature can be relatively easily described in terms of these two, making it a Kähler-Einstein space. Moreover, Q^n is the natural target space when considering the Gauss map of a hypersurface of a round sphere. We will discuss this relation -in particular for isoparametric hypersurfaces of spheres- and then study minimal Lagrangian submanifolds of Q^n, obtaining examples and some classifications, such as that of minimal Lagrangian submanifolds of Q^n with constant sectional curvature.

On equitorsion conformal and concircular transformations

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This is joint work with Professor Milan Zlatanovic. In this talk we will consider the independent curvature pseudotensors in GR_N. Next, we consider equitorsion conformal transformations of curvature pseudotensors of the first, second, third, sixth and seventh kind. The obtained values are called equitorsion conformal pseudotensors.

We will also consider equitorsion concircular transformations of curvature pseudotensors and determine the invariants of such transformations.

For the space GR_N and associated space R_N we give conformal tensor and concircular tensor from associated space R_N. Both are represented by means of independent curvature pseudotensor and torsion.

Shape and energies of geometric objects

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In this presentation, we want to introduce the manuscript of a book "Shape and Energies of Geometric Objects" whose authors are L. Kauffman, M. Najdanović, S. Rančić and me.

The primary motivation for this talk is to analyze application of Geometry and Topology in consideration of different objects and their shape.

In the first part we consider shape of deformable objects at small, infinitesimal deformations.

It is determined by their geometric curvature as well as curvature-based functionals which are said to be the energies of Geometric objects. Geometric objects that are discussed are curves surfaces and knots. To study energies and shape of curves and knots we use Willmore energy, total curvature, total torsion, total normalcy and Möbius energy. The examples are visualized.

The last part on "Knot Dynamics" is a self-contained research by Louis H Kauffman on the construction of examples of unknots and also knots that can be resistant to simplification by energy gradient and force evolution methods in programming models. We expect that our investigations of energy and Willmore energy will eventually shed light on such phenomena.

Universal method of obtaining invariants for geometric mapping

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In this paper, we presented which invariants for mappings of symmetric affine connection spaces may be obtained. This method was initially presented in (Vesić, 2020), and applied in many further research papers. In this paper, we will express invariants for mappings of symmetric affine connection spaces with respect to invariants of geodesic mappings.

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Volumes of polyhedrain hyperbolic 3-space with applications to the knot theory

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There are few volumeconjectures relating invariants of knots/links and 3-manifolds with their hyperbolicvolumes. We will consider the class of hyperbolic polyhedra with all dihedraangles equal to $\pi/2$. This class is very useful for construct hyperbolic3-manifolds and, in particular, link complements. There are will be presented upperbounds for volumes of polyhedra in terms of number of vertices obtained in arXiv:2111.08789. Some corollaries of these results will be discussed.

The curvature tensor of 6 dimensional nearly Kähler spaces

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We present a survey of how the curvature tensor of all known homogeneous 6 dimensional nearly Kähler spaces (both in the definite and in the pseudo Riemannian case) can be expressed in an invariant way using the induced structures on the 6 dimensionalnearly Kähler space. Having such an expression is a very usefull first step for studying the submanifolds of these spaces.

$\delta^\sharp(2,2)$-ideal hypersurfaces of dimension 4 in centroaffine differential geometry

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In centroaffine differential geometry, Chen's invariants denoted by $\delta^\sharp$ are crucial to determine an optimal bound for the squared norm of the Tchebychev vector field of a submanifold (For instance, see [1], [2], [4]-[7]. A submanifold attaining this bound is said to be an ideal centroaffine submanifold.

In this talk which is based on a joint work with Luc Vrancken, taking into account the theorem obtained in centroaffine differential geometry by adapting the proof of the theorem given in [3] related to curvature inequalities for Lagrangian submanifolds, we mention about the classifications of $\delta^\sharp(2,2)$-ideal centroaffine hypersurfaces of dimension 4.

Acknowledgements: This talk is based on some of the results of the joint work with Luc Vrancken obtained in the project numbered 33525 supported by the Scientific Research Projects Coordination Unit of Istanbul University.

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Some connections on non-symmetric gravitational theory

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We will consider a connection with totally anti-symmetric torsion on non-symmetric Riemann manifolds that satisfy Einstein's metric condition. We proved that an almost Hermitian manifold is a non-symmetric Riemann manifold that satisfies Einstein's metricity condition if and only if it is a Nearly Kahler manifold. We will also show what happens in the case of para Hermitian manifold, contact, and para-contact manifolds that satisfy Einstein's metricity condition.

This is joint paper with S. Ivanov.

The problem of envy-free division from the view point of algebraic topology

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Given some resource and a set of agents (players), one of the goals of welfare economics is to divide the resource among the agents in an envy-free manner. Envy-freeness, as a criterion of fair division, is the principle where every player feels that their share is at least as good as the share of any other agent, and thus no player feels envy. In a mathematical simplification a group of r players want to divide among themselves a commodity, commonly referred to as the "cake". The simplest model of the cake is the interval $[0, 1]$, which should be cut into r pieces by $r - 1$ cuts. The classical approach to envy-free division and equilibrium problems arising in mathematical economics typically relies on Knaster-Kuratowski-Mazurkiewicz

theorem, Sperner's lemma or some extension of these results involving mapping degree. We propose a different and relatively novel approach where the emphasis is on configuration spaces and equivariant topology, and the so called configuration space/test map scheme https://www.msri.org/workshops/378/schedules/2492, which was originally developed for applications in discrete and computational geometry and topology (Tverberg type problems, necklace splitting problem in the sense of N.Alon and D. West, etc.). We illustrate the method by proving several relatives (extensions) of the classical envy-free division theorem of David Gale, where the emphasis is on preferences allowing the players to choose degenerate pieces of the cake. We also show how this technique allows us to improve the splitting necklace theorem of N. Alon by adding natural constraints on the distribution of pieces of the necklace.

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